Список задач по построению коротковолновых асимптотик.

Первые десять задач каждого раздела предназначены для студентов физического отделения, следующие десять – для студентов химического отделения.

Уравнение Шредингера.

Построить коротковолновую асимптотику и указать интервал времени, на котором она определена, для следующей задачи:

1. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + 2x^2\psi, \quad \psi|_{t=0} = \sqrt{1-x^4}e^{\frac{2i}{h}x}.$

2. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + x^2\psi, \quad \psi|_{t=0} = \sqrt{4-x^2}e^{\frac{-i}{h}x}.$

3. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + 3x^2\psi, \quad \psi|_{t=0} = \sin(x)e^{\frac{i}{2h}x}.$

4. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + \frac{1}{2}x^2\psi, \quad \psi|_{t=0} = \sin(4-x)e^{\frac{3i}{h}x}.$

5. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi - x^2\psi, \quad \psi|_{t=0} = (x-2)e^{\frac{i}{h}x}.$

6. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (x-4)\psi, \quad \psi|_{t=0} = (2x-8)^2e^{\frac{i}{h}x^2}.$

7. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (2x-1)\psi, \quad \psi|_{t=0} = e^{-x^2}e^{\frac{i}{2h}x^2}.$

8. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (3-x)\psi, \quad \psi|_{t=0} = e^{-\frac{x^2}{2}}e^{\frac{2i}{h}x^2}.$

9. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (5+2x)\psi, \quad \psi|_{t=0} = \sqrt{3+x^2}e^{\frac{-i}{h}x^2}.$

10. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (1+x)\psi, \quad \psi|_{t=0} = \mathrm{tg}(x)e^{\frac{i}{h}(x-1)^2}.$

11. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (x^2+2x+1)\psi, \quad \psi|_{t=0} = \sqrt{9-4x^2}e^{\frac{i}{h}x}.$

12. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (4x^2-4x+1)\psi, \quad \psi|_{t=0} = \sqrt{4-x^4}e^{\frac{-2i}{h}x}.$

13. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (2x^2+8x+8)\psi, \quad \psi|_{t=0} = \sqrt{1+x^2}e^{\frac{i}{h}x}.$

14. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (x^2-6x+9)\psi, \quad \psi|_{t=0} = \mathrm{tg}(1-x)e^{\frac{i}{2h}x}.$

15. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi - (x^2+8x+16)\psi, \quad \psi|_{t=0} = \sin(2x)e^{\frac{-i}{3h}x}.$

16. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (3x-7)\psi, \quad \psi|_{t=0} = \cos(x)e^{\frac{i}{h}x^2}.$

17. $ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2}\Delta\psi + (2x+9)\psi, \quad \psi|_{t=0} = e^{-2x^2}e^{\frac{3i}{h}x^2}.$

18. $ih\dfrac{\partial\psi}{\partial t} = -\dfrac{h^2}{2}\Delta\psi + (1-4x)\psi, \quad \psi|_{t=0} = e^{-\frac{x^2}{2}} e^{\frac{-i}{h}x^2}.$

19. $ih\dfrac{\partial\psi}{\partial t} = -\dfrac{h^2}{2}\Delta\psi + (5+x)\psi, \quad \psi|_{t=0} = (x-2)^3 e^{\frac{i}{h}}.$

20. $ih\dfrac{\partial\psi}{\partial t} = -\dfrac{h^2}{2}\Delta\psi + (x^2+1)\psi, \quad \psi|_{t=0} = (2-x^3)e^{\frac{-i}{h}(x-1)^2}.$

Волновое уравнение.

Построить коротковолновую асимптотику для следующей задачи:

1. $u_{tt} - (x-3)^4 u_{xx} = 0, \quad \phi_0(x) = 2, \ S_0(x) = \dfrac{1}{3-x}.$

2. $u_{tt} - \dfrac{4}{(x+1)^2}u_{xx} = 0, \quad \phi_0(x) = 3\sqrt{x+1}, \ S_0(x) = \dfrac{1}{4}(x+1)^2.$

3. $u_{tt} - \dfrac{1}{(2x-1)^2}u_{xx} = 0, \quad \phi_0(x) = -\sqrt{x-\dfrac{1}{2}}, \ S_0(x) = (x-\dfrac{1}{2})^2.$

4. $u_{tt} - \dfrac{4}{x^4}u_{xx} = 0, \quad \phi_0(x) = x, \ S_0(x) = \dfrac{1}{6}x^3.$

5. $u_{tt} - 9x^6 u_{xx} = 0, \quad \phi_0(x) = \dfrac{4}{\sqrt{x^3}}, \ S_0(x) = -\dfrac{1}{6x^2}.$

6. $u_{tt} - (t+3)^4 u_{xx} = 0, \quad \phi_0(x) = 1, \ S_0(x) = 9 - x.$

7. $u_{tt} - 4(t+1)^2 u_{xx} = 0, \quad \phi_0(x) = -1, \ S_0(x) = 1 + x.$

8. $u_{tt} - \dfrac{1}{4}(t+2)^2 u_{xx} = 0, \quad \phi_0(x) = \dfrac{1}{2}, \ S_0(x) = 1 - x.$

9. $u_{tt} - \dfrac{4}{(t+1)^4}u_{xx} = 0, \quad \phi_0(x) = -\dfrac{1}{3}, \ S_0(x) = 2 + x.$

10. $u_{tt} - \dfrac{9}{(t+2)^6}u_{xx} = 0, \quad \phi_0(x) = 3, \ S_0(x) = \dfrac{3}{8} + x.$

11. $u_{tt} - (x+5)^6 u_{xx} = 0, \quad \phi_0(x) = \dfrac{1}{\sqrt{(x+5)^3}}, \ S_0(x) = -\dfrac{1}{2x^2+20x+50}.$

12. $u_{tt} - \dfrac{1}{(x-1)^4}u_{xx} = 0, \quad \phi_0(x) = 2x - 2, \ S_0(x) = \dfrac{1}{3}(x-1)^3.$

13. $u_{tt} - \dfrac{4}{(2x+3)^2}u_{xx} = 0, \quad \phi_0(x) = 2\sqrt{x+\dfrac{3}{2}}, \ S_0(x) = \dfrac{1}{2}(x+\dfrac{3}{2})^2.$

14. $u_{tt} - \dfrac{9}{x^6}u_{xx} = 0, \quad \phi_0(x) = 3\sqrt{x^3}, \ S_0(x) = \dfrac{1}{12}x^4.$

15. $u_{tt} - 4x^4 u_{xx} = 0, \quad \phi_0(x) = \dfrac{4}{x}, \ S_0(x) = -\dfrac{1}{2x}.$

16. $u_{tt} - (t+1)^6 u_{xx} = 0, \quad \phi_0(x) = 1, \ S_0(x) = -\dfrac{1}{4} + x.$

17. $u_{tt} - 4(t+2)^4 u_{xx} = 0, \quad \phi_0(x) = -1, \ S_0(x) = \dfrac{16}{3} - x.$

18. $u_{tt} - \dfrac{1}{4}(t+3)^6 u_{xx} = 0, \quad \phi_0(x) = \dfrac{1}{2}, \ S_0(x) = \dfrac{81}{8} - x.$

19. $u_{tt} - \dfrac{4}{(t+2)^4} u_{xx} = 0, \quad \phi_0(x) = -\dfrac{1}{3}, \ S_0(x) = -1 + x.$

20. $u_{tt} - \dfrac{9}{(t+1)^4} u_{xx} = 0, \quad \phi_0(x) = 3, \ S_0(x) = 3 + x.$